CS 490
Satisfiability Checking for Partially Adjacent Timed Propositional Temporal Logic is Decidable

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Preliminaries

- Timed Words: A timed word ρ over Σ is a finite sequence of pairs $(\sigma, \tau) \in \Sigma \times \mathbb{R}_{\geq 0}$ where σ are the propositions true at each timestamp and τ is the set of the timestamps.
- Metric Temporal Logic is a natural extension of Linear Temporal Logic with the primary difference being that we can associate timing intervals to the Until and Since modalities
- Timed Propositional Temporal Logic is another extension of LTL that uses freeze quantification to store the current timestamp. A freeze quantifier with clock variable x has the form $x.\phi$. When it is evaluated at a point i on a timed word, the timestamp τ_i at i is frozen or registered in x, and the formula ϕ is evaluated using this value of x.

  1-TPTL is the subclass of TPTL that uses only one clock variable.

Preliminaries

Rational Metric Temporal Logic:

- $Seg^+(\rho, x, y, \mathbf{S})$ is defined as untimed words $b_x, b_{x+1}...b_y$ in $\Gamma*$ for Γ = P[S] | S $\subset$ **S** such that for any x $\leq$ z $\leq$ y, b_z = P[W] iff ρ, z satisfies all the formulae in W and none of the formulae **S** $-$ W.
- $TSeg(\rho, i, I, \mathbf{S})$ is defined as $Seg^+(\rho, x, y, \mathbf{S})$ where $x > i$ and $\tau_x, \tau_y \in I + \tau_i$, and either $y = |dom(\rho)|$ or $\tau_{y+1} \notin I + \tau_i$.
- RatMTL:

  $\rho, i \models Rat_I re < \phi_1, ...\phi_n > \leftrightarrow TSeg(\rho, i, I, < \phi_1, ..., \phi_n >) \in L(re)$

  $\rho, i \models FRat_{I(} re < \phi_1, ..., \phi_n >, \phi) \leftrightarrow \exists j \geq i, \tau_j - \tau_i \in I, \rho, j \models \phi \wedge Seg^+(\rho, i+1, j-1, < \phi_1, ...\phi_n >) \in L(re)$

Preliminaries

Pneuli Extended Metric Temporal Logic:

The Future and Past Pneuli Automata modalities are defined as:

$\rho, i_0 \models \mathscr{F}^{k}_{I_1,\ldots,I_k}(A_k, \ldots, A_{k+1})(A)$ iff

$\exists i_0 \leq i_1 \leq i_2 \ldots \leq i_k \leq n$ s.t
$\wedge_{w=1}^{k}[(\tau_{i_w} - \tau_{i_0} \in I_w) \wedge Seg^{+}(\rho, i_{w-1}, i_w, S) \in L(A_w)] \wedge Seg^{+}(\rho, i_k, n, S) \in A_{k+1}$

$\rho, i_0 \models \mathscr{P}^{k}_{I_1,\ldots,I_k}(A_k, \ldots, A_{k+1})(A)$ iff

$\exists i_0 \geq i_1 \geq i_2 \ldots \geq i_k \geq 1$ s.t
$\wedge_{w=1}^{k}[(\tau_{i_0} - \tau_{i_w} \in I_w) \wedge Seg^{-}(\rho, i_{w-1}, i_w, S) \in L(A_w)] \wedge Seg^{-}(\rho, i_k, 1, S) \in A_{k+1}$

Preliminaries

Partially Adjacent 1-Timed Propositional Temporal Logic:

A set of intervals $I_1, I_2, ..., I_n$ is said to be partially adjacent if the positive side of the interval is allowed to be adjacent but the negative side is not. This is more formally defined as:

For any interval I_j:

$sup(I_j) > 0 \implies \text{T}$
$sup(I_j) < 0 \implies sup(I_j) \neq inf(I_k) \forall k$

The Sat-Checking Algorithm

Reduction of 1-PA TPTL to PA PnEMTL

The equisatisfiability of 1-PA TPTL and PA PnEMTL has already been established in Temporal Logic literature, specifically in S.N Krishna et al. - **From Non-punctuality to Non-adjacency: A Quest for Decidability of Timed Temporal Logics with Quantifiers** (2023)

Recall the definition of 1-PA TPTL. This naturally yields two kinds of modalities in the reduced PA PnEMTL formulae - $\mathscr{F}^{k}_{I_1,...,I_k}(A_k, ..., A_{k+1})(A)$ and $\mathscr{P}^{k}_{I_1,...,I_k}(A_k, ..., A_{k+1})(A)$.

The Future PnEMTL modality is unsrestricted due to the partially adjacent nature i.e intervals may overlap! However, the Past PnEMTL modality is restricted to not overlap.

Reduction of PA PnEMTL

To comment on decidability, we wish to now convert the PnEMTL modalities to a known decidable modality. Thoughts that come to mind regarding this are that the use of *oversampling projections* might help, and a reduction to **Partially Punctual Metric Temporal Logic** would be immenesely beneficial as well, due to it's structural similarity with Partial Adjacency and since it has already been proved as a decidable logic.

The proof of the above was shown by K. Madnani et al. - Partially punctual metric temporal logic is decidable (2016).

Reduction of PA PnEMTL

Therefore, on eliminating the F PnEMTL modality, we obtain punctual Until modalities and on doing the same for the P PnEMTL modality, we obtain non-punctual S modalities, which is exactly what PPMTL is!

The tricky part is showing the process of elimination of the F modalities. This is done by breaking up the individual conditions in the formula and expressing them using Rat and FRat.

For the elimination of the P modalities, we know with certainty that they are non-adjacent, and hence can be converted to a Past MITL modality, and this conversion is already well established in MTL literature.