

**Tutorial Sheet No. 5:
Applications of Integration**

- (1) Find the area of the region bounded by the given curves in each of the following cases.
- (i) $\sqrt{x} + \sqrt{y} = 1$, $x = 0$ and $y = 0$
 - (ii) $y = x^4 - 2x^2$ and $y = 2x^2$.
 - (iii) $x = 3y - y^2$ and $x + y = 3$
- (2) Let $f(x) = x - x^2$ and $g(x) = ax$. Determine a so that the region above the graph of g and below the graph of f has area 4.5
- (3) Find the area of the region inside the circle $r = 6a \cos \theta$ and outside the cardioid $r = 2a(1 + \cos \theta)$.
- (4) Find the arc length of the each of the curves described below.
- (i) the cycloid $x = t - \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$
 - (ii) $y = \int_0^x \sqrt{\cos 2t} dt$, $0 \leq x \leq \pi/4$.
- (5) For the following curve, find the arc length as well as the the area of the surface generated by revolving it about the line $y = -1$.
- $$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \leq x \leq 3$$
- (6) The cross sections of a certain solid by planes perpendicular to the x -axis are circles with diameters extending from the curve $y = x^2$ to the curve $y = 8 - x^2$. The solid lies between the points of intersection of these two curves. Find its volume.
- (7) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $y^2 + z^2 = a^2$.
- (8) A fixed line L in 3-space and a square of side r in a plane perpendicular to L are given. One vertex of the square is on L . As this vertex moves a distance h along L , the square turns through a full revolution with L as the axis. Find the volume of the solid generated by this motion.
- (9) Find the volume of the solid generated when the region bounded by the curves $y = 3 - x^2$ and $y = -1$ is revolved about the line $y = -1$, by both the Washer Method and the Shell Method.
- (10) A round hole of radius $\sqrt{3}$ cms is bored through the center of a solid ball of radius 2 cms. Find the volume cut out.
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