MA 109 Tutorial Batch D1 T2
Recap 2

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https://agnipratimnag.github.io/ma109/

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Some things you might have missed...

First, please take out your phones and join the MA 109 Google Classroom if you haven't already. The code is **aegghdl**. Use your personal Google accounts, not LDAP.

All course resources can be found at
`https://agnipratimnag.github.io/ma109/`

I have also put up a feedback form. Anything you think I could improve on, or do differently during my tuts, please feel free to let me know. The form is anonymous, so say anything and everything you want to.

Okay, let's get started!

Continuity

Cute JEE definition:
We can draw the function without taking the pen off the paper.

The adult version of the above statement is this:

Definition ($\epsilon - N_0$ definition of continuity)

A function f : $\mathbb{R} \to \mathbb{R}$ is called continuous at a point p $\in \mathbb{R}$ if for every sequence $(x_n)_{n=1}^{\infty}$ that converges to p, the sequence $(f(x)_n)_{n=1}^{\infty}$ converges to f (p).

Definition ($\epsilon - \delta$ definition of continuity)

A function f : $\mathbb{R} \to \mathbb{R}$ is called continuous at a point p $\in \mathbb{R}$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - f(p)| < \epsilon$ for every $|x - p| < \delta$.

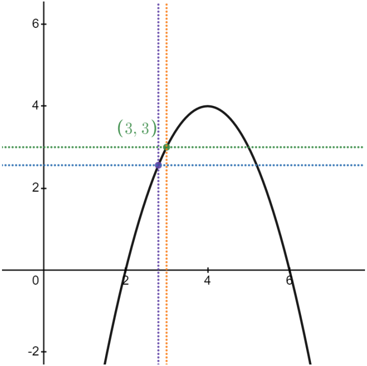
Visualising!

Everytime I give you an ϵ neighbourhood for the output of the function you should be able to give me a δ neighbourhood for the input of the function **if** you claim that it is a continuous function.

For this parabola, if we examine continuity about the point (3,3), are you able to notice what the ϵ neighbourhood looks like?
It's the green minus blue distance on the Y axis.

The corresponding δ neighbourhood is the orange minus purple distance on the X axis.

Check it out yourself over here!



Time to actually do a proof.

Show that the function $ln(\frac{1}{1+x})$ is continuous at $x_0 = 0$ by the $\epsilon - \delta$ definition of continuity.

Limits

The definition of a limit is very similar to that of continuity. The only difference is that the number the function converges to need not be the value of the function at that point.

Definition ($\epsilon - N_0$ definition of a limit)

Let U be an open subset of $\mathbb{R}$ and let $c \in U$. A function f: U $\setminus\{$c$\}$ $\rightarrow \mathbb{R}$ is said to have a limit ℓ at a point c if for every sequence $(x_n)_{n=1}^{\infty}$ that converges to c, the sequence $(f(x)_n)_{n=1}^{\infty}$ converges to ℓ.

Definition ($\epsilon - \delta$ definition of a limit)

Let U be an open subset of $\mathbb{R}$ and let $c \in U$. A function f: U $\setminus\{$c$\}$ $\rightarrow \mathbb{R}$ is said to have a limit ℓ at a point c if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - \ell| < \epsilon$ for every $|x - c| < \delta$.

To check left and right hand limits, follow the same definition but with the slight tweak that your input to the function is strictly less than or greater than c.

Differentiability

Define two things.

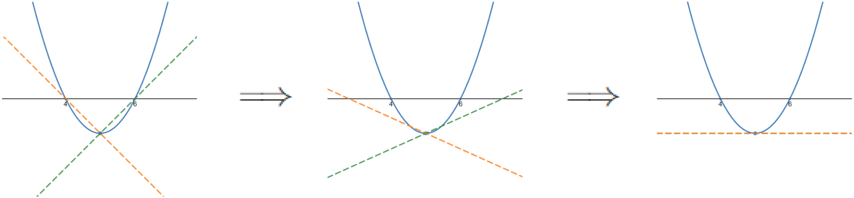
- the **r-neighbourhood** of a point t_0, referred to as $B_r(t_0)$ which consists of all t such that $|t - t_0| < r$.
- the function $d_t(\text{f}) = \frac{f(x+t)-f(x)}{t}$ for $|t| < \epsilon$ for some positive ϵ and let t $\rightarrow$ 0. Think of this function as $\frac{\textbf{change in output}}{\textbf{change in input}}$.

Definition (Differentiability)

A function f : U $\rightarrow \mathbb{R}$ is said to be differentiable at a point p $\in$ U if there exists an $\epsilon > 0$ such that the function $d_t(\text{f}) : B_\epsilon(0) \rightarrow \mathbb{R}$ has a limit at 0.

What?

As you approach a point from the left and right, the slope of the orange secant and the slope of the green secant should tend to the same limit.



To understand this better, you can see the graph in motion here!

Some properties...

- Differentiability $\implies$ Continuity
- Discontinuity $\implies$ Non-differentiable
- Continuity $\centernot\implies$ Differentiability