2.9. Tutorial Sheet No. 8: Multiple Integrals

(1) For the following, write an equivalent iterated integral with the order of integration reversed:

(i) $\int_0^1 \left[\int_1^{e^x} dy\right] dx.$ (ii) $\int_0^1 \left[\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dx\right] dy.$

(2) Evaluate the following integrals:

(i) $\int_0^{\pi} \left[\int_x^{\pi} \frac{\sin y}{y} dy\right] dx.$ (ii) $\int_0^1 \left[\int_y^1 x^2 e^{xy} dx\right] dy.$

(iii) $\int_0^2 (\tan^{-1} \pi x - \tan^{-1} x)dx.$

(3) Find $\iint_D f(x,y)d(x,y)$, where $f(x,y) = e^{x^2}$ and D is the region bounded by the lines $y = 0,\ x = 1$ and $y = 2x$.

(4) Evaluate the integral

$$\iint_D (x-y)^2 \sin^2(x+y)d(x,y),$$

where D is the parallelogram with vertices at $(\pi, 0),\ (2\pi, \pi),\ (\pi, 2\pi)$ and $(0, \pi)$.

(5) Let D be the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy = 1,\ xy = 9$ and the lines $y = x,\ y = 4x$. Find $\iint_D d(x,y)$ by transforming it to $\iint_E d(u,v)$, where $x = \frac{u}{v},\ y = uv,\ v > 0.$

(6) Find

$$\lim_{r\to\infty} \iint_{D(r)} e^{-(x^2+y^2)} d(x,y),$$

where $D(r)$ equals:

(i) $\{(x,y) : x^2 + y^2 \le r^2\}.$
(ii) $\{(x,y) : x^2 + y^2 \le r^2,\ x \ge 0,\ y \ge 0\}.$
(iii) $\{(x,y) : |x| \le r,\ |y| \le r\}.$
(iv) $\{(x,y) : 0 \le x \le r,\ 0 \le y \le r\}.$

(7) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ using double integral over a region in the plane. (Hint: Consider the part in the first octant.)

(8) Express the solid $D = \{(x,y,z) | \sqrt{x^2+y^2} \le z \le 1\}$ as

$$\{(x,y,z) | a \le x \le b, \phi_1(x) \le y \le \phi_2(x), \xi_1(x,y) \le z \le \xi_2(x,y)\}.$$

(9) Evaluate

$$I = \int_0^{\sqrt{2}} \left(\int_0^{\sqrt{2-x^2}} \left(\int_{x^2+y^2}^{2} x dz \right) dy \right) dx.$$

Sketch the region of integration and evaluate the integral by expressing the order of integration as $dxdydz$.

(10) Using suitable change of variables, evaluate the following:

(i)

$$I = \iiint_D (z^2x^2 + z^2y^2) dxdydz$$

where D is the cylindrical region $x^2+y^2 \leq 1$ bounded by $-1 \leq z \leq 1$.

(ii)

$$I = \iiint_D \exp(x^2 + y^2 + z^2)^{3/2} dxdydz$$

over the region enclosed by the unit sphere in $\mathbb{R}^3$.