2.15. Tutorial Sheet No.14: Stoke's theorem and applications

(1) Consider the vector field $\mathbf{F} = (x-y)\mathbf{i} + (x+z)\mathbf{j} + (y+z)\mathbf{k}$. Verify Stokes theorem for $\mathbf{F}$ where S is the surface of the cone: $z^2 = x^2 + y^2$ intercepted by
(a) $x^2 + (y-a)^2 + z^2 = a^2 : z \geq 0$ (b) $x^2 + (y-a)^2 = a^2$

(2) Evaluate using Stokes Theorem, the line integral

$$\oint_C yz\,dx + xz\,dy + xy\,dz,$$

where C is the curve of intersection of $x^2 + 9y^2 = 9$ and $z = y^2 + 1$ with clockwise orientation when viewed from the origin.

(3) Compute

$$\iint_S (\mathrm{curl}\,\mathbf{v}) \cdot \mathbf{n} dS,$$

where $\mathbf{v} = y\mathbf{i} + xz^3\mathbf{j} - zy^3\mathbf{k}$ and $\mathbf{n}$ is the outward unit normal to S, the surface of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = -3$.

(4) Compute $\oint_C \mathbf{v} \cdot d\mathbf{r}$ for

$$\mathbf{v} = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2},$$

where C is the circle of unit radius in the xy plane centered at the origin and oriented clockwise. Can the above line integral be computed using Stokes Theorem?

(5) Compute

$$\oint_C (y^2 - z^2)dx + (z^2 - x^2)dy + (x^2 - y^2)dz,$$

where C is the curve cut out of the boundary of the cube

$$0 \leq x \leq a,\ 0 \leq y \leq a,\ 0 \leq z \leq a$$

by the plane $x + y + z = \frac{3}{2}a$ (specify the orientation of C.)

(6) Calculate

$$\oint_C ydx + zdy + xdz,$$

where C is the intersection of the surface $bz = xy$ and the cylinder $x^2 + y^2 = a^2$, oriented counter clockwise as viewed from a point high upon the positive z-axis.

(7) Consider a plane with unit normal $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. For a closed curve C lying in this plane, show that the area enclosed by C is given by

$$A(C) = \frac{1}{2}\oint_C (bz - cy)dx + (cx - az)dy + (ay - bx)dz,$$

where C is given the anti-clockwise orientation. Compute $A(C)$ for the curve C given by

$$\mathbf{u}\cos t + \mathbf{v}\sin t,\ 0 \leq t \leq 2\pi.$$

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