PH 111 D1 T4
Recap 1

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March 15, 2023

Welcome!

Hello everyone. A very warm welcome to PH 111 to all of you. In this course you'll have a quick recap of some JEE concepts, and learn how to use them in some slightly more complicated scenarios. After that, we'll move on to Special Relativity, before starting Quantum Mechanics post-midsem.

(Maybe you'll understand Interstellar better after this semester)

As you might know, this is a completely new course running for the first time, so you do not have the luxury of things such as past papers and previous year resources. However, we will do our best to provide you with sufficient amount of resources so that you have enough things to work on.

From your side, do your best to diligently attend classes and tutorials. Be active in getting your doubts cleared. Feel free to text me for any doubts or pester me for as long as you want in class. Sounds good?

All course resources will be uploaded at https://agnipratimnag.github.io/ph111/

Let's get started!

Basics

These are the basic ideas covered in the first chapter of the course.

I think you have done most of this during JEE quite rigorously so I won't recap them formally, but let me know if there is something you have a doubt in.

- Vectors:
  - Vector addition, dot and cross products.
- Cartesian Coordinate System:
  - Identifying the basis vectors and how they are evaluated in dot and cross products.
  - Position vector and infinitesimal displacement vector.
- Plane Polar Coordinate System
  - Understanding the relation between this and the Cartesian system.
  - Geometrically understanding the transformation equation.
  - Note that in this system, the basis vectors keep changing based on your location, which does not happen in Cartesian.
  - If you were moving on a circle, your position vector would be $r\hat{r}$ throughout. Isn't this odd, you're moving around but your position vector stays the same?

Kinematics

Again, we look at things in Cartesian as well as Plane Polar.

In Cartesian, it's the most basic equations of kinematics (who is whose derivative or integral) coupled with basic math, and you have your equations of motion.

In plane polar, things might become a little bit more complex. To start with, if we try finding $\frac{d\hat{r}}{dt}$, we have to first convert to Cartesian, and then differentiate. This is done so that the basic vectors are fixed in time and we don't have to differentiate that as well.

$$\mathbf{r} = r\hat{r}$$

$$\mathbf{v} = v_r\hat{r} + v_\theta\hat{\theta}$$

where v_r is radial velocity $= \dot{r}$ and v_θ is tangential velocity $= r\dot{\theta}$

Consider motion on a circle, motion from smaller circles to larger circles, and random motion as well maybe. See if you're able to somewhat grasp what these quantities look like in those situations.

The ugly cousin of $\ddot{x}$

We have figured out what velocity and position look like in both systems. Naturally, we move on to acceleration now. Cartesian is simple, so Plane Polar is where go to again.

$$\mathbf{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\mathbf{a} = \frac{d}{dt}\ \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

(product rule time)

$$\mathbf{a} = \ddot{r}\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\hat{\theta}}{dt}$$

Find $\frac{d\hat{\theta}}{dt}$ using the hint I gave you earlier. That should yield:

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

And we're done!

Breaking it down

Each component has a meaning of its own.

- $\ddot{r}$: Simple radial acceleration - when you acceleration along the radial direction only, this term will account for that.
- $r\dot{\theta}^2$: Centripetal acceleration - when you're undergoing uniform circular motion, the net force on you must be the centripetal force, which acts radially inwards.

  Note that I am isolating cases where only that term shows up, and the rest are zero so that you are able to understand them individually. In more realistic situations, things are more complicated and multiple accelerations are present together.
- $r\ddot{\theta}$: Angular acceleration - when you move on a circle, and your ω is increasing constantly at some rate, this term captures that.
- $2\dot{r}\dot{\theta}$: Coriolis acceleration - a fake acceleration which comes into play when we are sitting in rotating coordinate systems. To gather some intuition for now, you can watch this.