PH111: Tutorial Sheet 2

This tutorial sheet contains problems related to plane-polar coordinate system.

1. Using known results from the Cartesian coordinate system, and the relation between plane-polar and Cartesian basis vectors, calculate: (a) $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}}$, (b) $\hat{\boldsymbol{\theta}} \times \hat{\mathbf{k}}$, and (c) $\hat{\mathbf{k}} \times \hat{\mathbf{r}}$.

2. A particle is moving along a circular path of radius a, with angular velocity given by $\omega(t) = \omega_0 + \alpha t$, where ω_0 and α are constants. Obtain the radial and tangential components of its velocity and acceleration.

3. A particle is moving along the line $y = a$, with the velocity $\mathbf{v} = u\hat{\mathbf{i}}$, where u is a constant. Express its velocity in plane polar coordinates.

4. A particle moves in such a way that $\dot{\theta} = \omega$ (constant), and $r = r_0 e^{\beta t}$, where r_0 and β are constants. Write down its velocity and acceleration in plane polar coordinates. For what values of β will the radial acceleration of the particle by zero?

5. Consider a circle of radius a, with the origin of the plane polar coordinate system placed at a point on the circumference. The particle is moving along the circle with a constant speed u.

   (a) What is the equation of the circle in this coordinate system?

   (b) What is the value of $\dot{\theta}$ in terms of u and a?

   (c) Write down the velocity of the particle in plane-polar coordinate system.

   (d) What is the acceleration of the particle in plane-polar coordinate system?

6. A particle moves along the curve $r = A\theta$, with $A = 1/\pi$ m/rad, and $\theta = \alpha t^2$, where α is a constant. Obtain the expressions for the velocity and acceleration of this particle in plane polar coordinates.

   (a) Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.

   (b) At what angles do radial and tangential components of the acceleration have equal magnitude?