

PH111: Tutorial Sheet 3

This tutorial sheet contains problems related to work-energy theorem, conservative force, and potential energy.

- * Mass m rotates on a frictionless table, held to circular path by a string which passes through a hole in the table. The string is slowly pulled through the hole so that the radius of the circle changes from R_0 to R_1 . Show that the work done in pulling the string equals the increase in kinetic energy of the mass.
- * A particle of mass m moves in one dimension along the positive x axis. It is acted on by a constant force directed towards the origin with the magnitude B , and an inverse law repulsive force of magnitude A/x^2 .
 - Find the potential energy function $V(x)$
 - Plot the potential energy as a function of x , and the total energy of the system, assuming that the maximum kinetic energy is $K_0 = \frac{1}{2}mv_0^2$.
 - What is the point of equilibrium, i.e., the point where net force acting on the particle is zero.
- * A particle of mass M is held fixed at the origin. The gravitational potential energy of another particle of mass m , in the field of the first mass, is given by

$$V(\mathbf{r}) = -\frac{GMm}{r},$$

where G is the gravitational constant, and r is the distance of mass m from the origin.

- What is the force acting on the particle of mass m ?
 - Calculate the curl of this force.
- * Consider a 2D force field $\mathbf{F} = A(y^2\hat{\mathbf{i}} + 2x^2\hat{\mathbf{j}})$. Calculate the work done by this force in going around a closed path which is a square made up of sides of length a , lying in the xy -plane, with two of its vertices located at the origin, and point (a, a) . Find the answer by doing the line integral, as well as by using the Stokes' theorem. The path is traversed in a counter clockwise manner.
 - Find the forces for the following potential energies
 - $V(x, y, z) = Ax^2 + By^2 + Cz^2$
 - * $V(x, y, z) = A \ln(x^2 + y^2 + z^2)$
 - * $V(r, \theta) = A \cos \theta / r^2$ (r and θ are plane polar coordinates)

Above, A , B , and, C are constants.

- Determine whether each of the following forces is conservative. Find the potential energy function if it exists. A , α , β are constants.

(a) $\ast \mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

(b) $\ast \mathbf{F} = Axyz(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

(c) $F_x = A \sin(\alpha y) \cos(\beta z)$, $F_y = -Ax\alpha \cos(\alpha y) \cos(\beta z)$, $F_z = Ax \sin(\alpha y) \sin(\beta z)$