PH 207, Introduction to Special Theory of Relativity: Final examination

Total marks: 60; Time allowed: 2 hours
20 September 2022

- You are allowed to use one A4-sized formula-sheet that you have prepared for yourself.

1. [3+3+3+3+3=15 marks] State whether the following statements are true or false. Support your answer with appropriate calculation or reasoning—without this you will not be awarded any marks even if your answer is correct.
   (a) Suppose two successive Lorentz transformations are made, first for a relative velocity in the x-direction and then for a relative velocity in the y-direction. If we switch the order of these transformations, the combined transformation will be different.
   (b) The quantity $E^2 + c^2B^2$ is invariant under a Lorentz transformation.
   (c) A force applied in a certain direction on a relativistic object can accelerate the object in a perpendicular direction.
   (d) Since time is a frame-dependent quantity in relativistic mechanics, if event A *leads to* event B at a later time in one frame, there exists a frame in which event B can happen before event A.
   (e) Suppose in a certain frame there is an electric and a magnetic field. It is always possible to find a frame where either of the fields vanishes.
2. [15 marks] Suppose a particle of mass m and charge q is subject to a homogeneous electric field $\mathbf{E}$. We want to find the trajectory of the particle, i.e., its position $\mathbf{r}$ as a function of time t. In class we solved this problem assuming that the particle starts from rest at the origin at time $t = 0$. In this problem we will change the initial conditions: assume that the particle is at $\mathbf{r}_0$ with momentum $\mathbf{p}_0$ at $t = 0$. Calculate $\mathbf{r}(t)$.
3. [20 marks] We continue with the previous problem, but now include a uniform magnetic field $\mathbf{B}$ in addition to the uniform electric field $\mathbf{E}$, with the two fields perpendicular to each other. Find $\mathbf{r}(t)$. For simplicity, assume that the charge starts from rest at the origin at $t = 0$, and $E < cB$. [*Hint*: Go to a frame where $E = 0$. Find the trajectory in that frame. Transform back to the original frame.]
4. [10 marks] Consider a relativistic binary collision process where a particle of rest mass m_{0a} collides with a target stationary particle of rest mass m_{0b} to produce a bunch of particles each of rest mass m_{oi}, $i = 1$ to N. Such processes are quite common in particle-physics. Show that the threshold energy of the incoming particle for this process to happen is given by

$$E_a|_{\text{threshold}} = \frac{\left(\sum m_{0i}\right)^2 - m_{0a}^2 - m_{0b}^2}{2m_{0b}}. \tag{1}$$